Taking into account Eq. (4), Eq. (10) represents $m(m-1)/2$ linear connections between the constraints given in Eq. (3). In other words, Eq. (3) represents only $nm-m(m-1)/2$ independent equations and therefore $\lambda_y(m\times n)$ must have exactly the same number of independent coefficients. It is interesting to note that when X contains the full set of mode shapes $(m=n)$, the number of independent equations in Eq. (3) becomes equal to the number of independent variables, $n(n+1)/2$, which is in full agreement with the physical situation. In this case one obtains from Eq. (9)[2,3] or directly from Eq. (3)

$$Y_{\text{full}}=MX_{\text{full}}\Omega^2_{\text{full}}X^t_{\text{full}}M \tag{11}$$

Now, any set of connections between the coefficients of λ_y which reduces the number of independent coefficients to the number of independent equations in Eq. (3) and does not violate Eq. (10) is permissible. Equation (8) represents such a permissible set. The mathematical and physical meaning of Eq. (8) is now clear: it represents $m(m-1)/2$ linear connections between the coefficients of λ_y which take into account the orthogonality conditions between the mode shapes. Equation (8) makes the problem well defined.

Conclusion

It was shown that the assumption $\lambda^t_y MX=X^t M\lambda_y$ used in Ref. 3 to obtain a corrected stiffness matrix has clear mathematical and physical meaning. It reduces the number of coefficients in the Lagrange multiplier λ_y to the number of independent equations in full consideration with the orthogonality conditions.

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Generalized Substructure Coupling Procedure for Damped Systems

Roy R. Craig Jr.* and Yung-Tseng Chung†
The University of Texas, Austin, Tex.

Introduction

MANY previous papers have discussed the use of substructure coupling, in particular the version of substructure coupling referred to as component mode synthesis, for solving large structural dynamics problems. Only a very limited number of these works have treated damped structures, and the usual approach has been to assume "proportional damping" and to assume the structures to be lightly damped. However, there are important instances in which these damping assumptions are invalid. For example, structures with active control systems, with concentrated dampers, or with rotating parts fall in this category. Also, it is quite common for structural modes that are measured experimentally to be complex modes, i.e., modes which do not satisfy the proportional damping assumption. In Ref. 1 Hasselman and Kaplan extended the Craig-Bampton form of the classical Hurty method of substructure coupling to systems with nonproportional damping by using fixed-interface complex substructure modes. A generalized substructure coupling procedure for damped structures was presented in Ref. 2. The present Note summarizes the findings of that work by presenting a generalized substructure coupling procedure for damped systems. The essential ingredients of the method are 1) a Hamiltonian first-order differential equation formulation is used, and 2) a generalized coupling procedure is employed, permitting all significant types of constraints and coordinate transformations to be invoked. An alternative state vector formulation is presently under study.

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*Professor, Aerospace Engineering and Engineering Mechanics. Member AIAA.

†Graduate Student, Engineering Mechanics.

Theoretical Formulation

The name "component mode synthesis" is applied to substructure coupling procedures in which the motion of each substructure is represented by a selected set of component modes. Equations of interface compatibility are employed to obtain an independent set of system equations of motion. In the present Note, the substructure equations are written in the first order form

$$a\dot{y}+by=f \tag{1}$$

where

$$a=\begin{bmatrix} 0 & I \\ I & c \end{bmatrix},\quad b=\begin{bmatrix} -m^{-1} & 0 \\ 0 & k \end{bmatrix},\quad y=\begin{Bmatrix} p \\ x \end{Bmatrix},\quad f=\begin{Bmatrix} 0 \\ f_x \end{Bmatrix} \tag{2}$$

where x is the substructure physical coordinate vector, $p=m\dot{x}$ the substructure momentum vector, m the substructure mass matrix, c the substructure damping matrix, and k the substructure stiffness matrix. [The form of a and b in Eq. (2) is selected so that the matrices will be symmetric when c is symmetric.] Substructure modes, which may be complex, are obtained by solving the homogeneous form of Eq. (1) by assuming a solution of the form

$$y=\psi e^{\lambda t} \tag{3}$$

For underdamped systems the eigenvalues λ_r and eigenvectors ψ_r occur in complex conjugate pairs. Let Ψ be the collection of substructure modal vectors. Then the modal synthesis coordinate transformation is given by

$$y=\Psi z \tag{4}$$

where z is the vector of substructure modal coordinates.

When a structure consists of several substructures, the coordinates x will be subject to interface compatibility conditions (constraints). Let X, P, and Y be the collection of all substructure x, p, and y coordinates, respectively. Thus,

$$P=M\dot{X} \tag{5}$$

Table 1 Frequency and damping errors for a clamped-clamped beam with nonproportional damping ($N_\alpha = 12, N_\beta = 8, N_t = 18$ is reference solution)

Mode	$N_\alpha = 10, N_\beta = 8, N_t = 16$ σ error, %	ω error, %	$N_\alpha = 8, N_\beta = 6, N_t = 12$ σ error, %	ω error, %	$N_\alpha = 6, N_\beta = 6, N_t = 10$ σ error, %	ω error, %
1	1.88	1.17	5.31	3.09	6.37	3.97
2	1.24	0.81	5.62	2.70	6.64	3.21
3	0.06	0.65	0.24	1.42	0.19	2.11
4	1.52	2.01	0.67	6.46	2.81	8.30
5	0.15	0.01	2.85	0.30	1.27	0.35
6	4.84	2.18	20.15	5.76	26.74	8.36
7	6.98	0.68	22.87	4.06	31.59	4.61
8	8.75	1.10	20.45	2.19	31.34	4.27
9	3.89	2.67	0.58	11.99	54.71	18.76
10	5.50	0.27	5.63	0.18	24.52	50.80
11	10.22	3.47	54.66	8.08	—	—
12	7.91	0.05	31.70	7.76	—	—
13	17.66	2.09	—	—	—	—
14	10.49	4.27	—	—	—	—
15	12.73	0.94	—	—	—	—
16	9.11	11.39	—	—	—	—
17	—	—	—	—	—	—
18	—	—	—	—	—	—

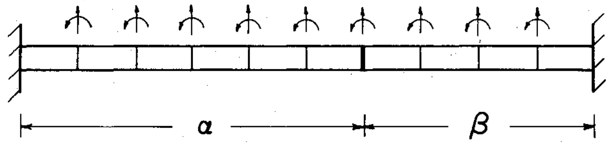

Fig. 1 Eighteen degree of freedom clamped-clamped beam.

where M has substructure mass matrices along its principal diagonal.

Let the coordinates X be subject to C linear constraint equations given by

$$E_I X = 0 \tag{6}$$

Then the momentum vector P will be subject to the constraints

$$E_I \dot{X} = E_I M^{-1} P = 0 \tag{7}$$

Equations (6) and (7) can be collected to form the constraint equation

$$EY = 0 \tag{8}$$

where

$$E = \begin{bmatrix} E_I M^{-1} & 0 \\ 0 & E_I \end{bmatrix} \tag{9}$$

One goal of component mode synthesis is to reduce the number of coordinates required to describe the dynamics of a system. This is done by truncating the set of modal coordinates. Let $\hat{\Psi}$ and $\hat{Z}$ be the truncated matrix of modal vectors and the truncated vector of modal coordinates, respectively. Then Y is approximated by the coordinate transformation

$$Y = \hat{\Psi}\hat{Z} \tag{10}$$

Equations (8) and (10) can be combined to give the constraint equation

$$G\hat{Z} = 0 \tag{11}$$

The standard procedure for enforcing Eq. (11) is to define an independent set of system coordinates U such that

$$\hat{Z} = SU \tag{12}$$

where

$$S = \begin{bmatrix} -G_{dd}^{-1} G_{d\ell} \\ I_{\ell\ell} \end{bmatrix} \tag{13}$$

where G has been partitioned into a nonsingular square submatrix G_{dd} and a matrix $G_{d\ell}$, i.e.,

$$G = [G_{dd} G_{d\ell}] \tag{14}$$

It is shown in Ref. 2 that, due to orthogonality and normalization of the substructure modes, the final set of independent system equations of motion for the damped, coupled system can be written in the form

$$(S^T S)\dot{U} - (S^T \hat{\Lambda} S) U = S^T \hat{\Psi} F \tag{15}$$

where $\hat{\Lambda}$ is a diagonal matrix of eigenvalues corresponding to the modes in $\hat{\Psi}$.

The system equations, Eqs. (5-15), can be written in more explicit substructure formulation. For example, let α and β be the labels of two substructures that form a coupled system. Then, let

$$Y = [p^\alpha x^\alpha p^\beta x^\beta]^T \tag{16}$$

Then,

$$E = [E^\alpha E^\beta] \tag{17}$$

$$G = [E^\alpha \hat{\Psi}^\alpha \, E^\beta \hat{\Psi}^\beta] \tag{18}$$

and so forth.

Example Problem

In Ref. 2 two example problems are cited: a truss problem and a beam problem. Only the results of the beam problem will be given here. In this example only free-interface substructure modes are used. It is well known[3] that, unless supplemented by residual flexibility modes, free-interface normal modes do not give good convergence in substructure coupling applications. Research currently is underway on the use of other forms of substructure modes, e.g., fixed-interface normal modes and residual flexibility (static) modes.

Figure 1 shows a clamped-clamped uniform beam represented by ten finite elements and divided into two substructures α and β. In order that the system not have proportional damping it is assumed that

$$c^\alpha = (1/48)k^\alpha, \qquad c^\beta = (1/96)k^\beta \tag{19}$$

Comparisons are made between modal synthesis approximations and reference solutions for system eigenvalues and eigenvectors. The complex eigenvalues are denoted by

$$\lambda = -\sigma + i\omega \tag{20}$$

and, in Table 1, the percentage error in σ and ω is given for three levels of truncation.

From the results in Table 1 it can be seen that a truncated set of free-interface substructure complex modes can be employed to represent a system with nonproportional damping by using the proposed first-order substructure coupling method. It is of interest to note that the frequency and damping errors are relatively large for many modes. This is probably due to the fact that only free-interface complex modes were employed. Also it is of interest to note that frequency and damping convergence is not necessarily monotonic (e.g., mode 3 damping) as in the familiar Rayleigh-Ritz approximation of frequencies of undamped systems.

Concluding Remarks

A generalized substructure coupling procedure for damped structures has been described, and a typical example of the effect of modal truncation on system eigenvalues has been presented. Further research on related coupling methods is currently in progress.

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Torsional Vibrations of Orthotropic Conical Shells

J. S. Tomar*
University of Roorkee, Roorkee, India
D. C. Gupta†
J. V. Jain Postgraduate College, Saharanpur, India
and
R. K. Sharma‡
Central Building Research Institute, Roorkee, India

Introduction

A SIMPLE model is presented for use by researchers in engineering and physical sciences working specifically on the design of modern missiles and space vehicles. With the advancement of space research, it has become necessary to obtain a greater insight into the behavior of shells of orthotropic material which are frequently used in missiles and other allied systems. In spite of the importance of the problem, little work has been done on such shells, although Hearmon and Mirsky[1] and Garnet et al.[2,3] have presented a study of torsional vibrations of orthotropic cylindrical shells.

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*Professor, Department of Applied Mathematics.
†Lecturer, Department of Mathematics.
‡Scientist, Planning Division.

In this Note the differential equation governing the torsional vibrations of an orthotropic conical shell is obtained and solved by the Rayleigh-Ritz technique of assuming the displacements as an infinite series in terms of meridional coordinates. Numerical results for the frequency parameter have been computed for the first two modes of vibration for different values of length parameter, thickness parameter, and semivertical angle.

Deformations

Consider a conical shell of axial length ℓ, thickness h, and semivertical angle α and let R be the mean cross-sectional radius of the shell. Let the shell be referred to a coordinate system r, θ, z, where r is the radial coordinate measured from the middle surface of the shell along the outward drawn normal to the surface, θ the tangential direction, and z the meridional direction. Let u_r, u_θ, and u_z be the components of displacement in r, θ, and z directions, respectively. Since the torsional vibrations are being considered here for the shell,

$$u_r = 0 = u_z \text{ and } \partial(\ )/\partial\theta = 0 \tag{1}$$

The displacement u_θ is approximated by $\bar{u}_\theta$ as

$$\bar{u}_\theta = v + r\Psi_\theta \tag{2}$$

where v is the tangential displacement in the direction of θ of any particle which is lying on the middle surface of the shell and Ψ_θ is the rotation of a normal element in the $r\theta$ plane.

Moment and Force Resultants

Moment and force resultants as defined[4] reduce to

$$N_{z\theta} = C_{66} h\left(\frac{\partial v}{\partial z} + \frac{h^2}{12R_1}\frac{\partial \Psi_\theta}{\partial z}\right)$$
$$M_{z\theta} = C_{66}\frac{h^3}{12}\left(\frac{1}{R_1}\frac{\partial v}{\partial z} + \frac{\partial \Psi_\theta}{\partial z}\right) \tag{3}$$

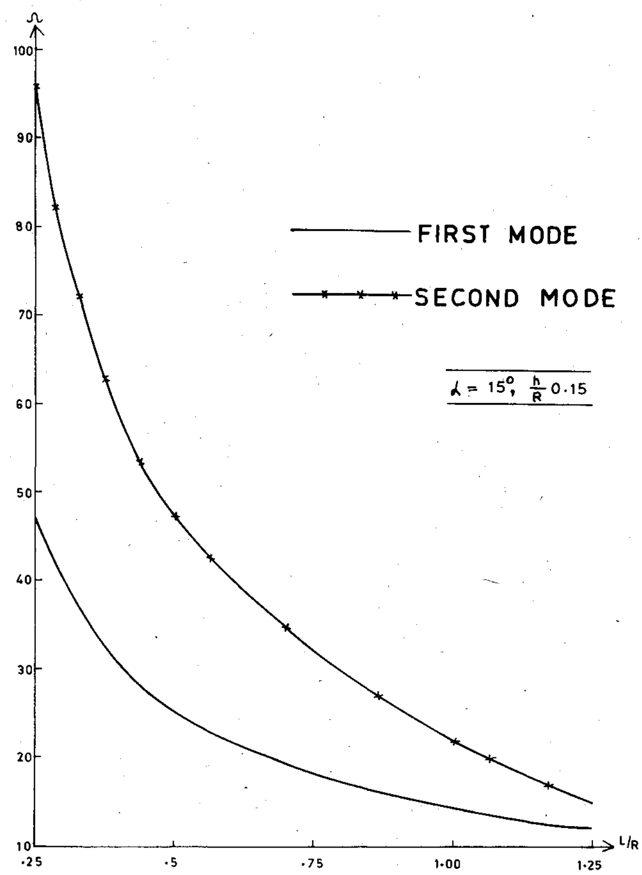

Fig. 1 Variation of frequency with length parameter for clamped orthotropic conical shells in the first two modes of vibration.